COBEM 99
XV Congresso Brasileiro de Engenharia Mecânica
15th Brazilian Congress of Mechanical Engineering
22 - 26 de Novembro de 1999 / November 22 - 26, 1999 Águas de Lindóia, São Paulo,

PREDICTING THE BEHAVIOR OF THE WAKE OF A RECTANGULAR CYLINDER FROM EXPERIMENTAL CHAOTIC TIME SERIES

Tito Dias Júnior
Cláudio Lindquist
Emanuel Rocha Woiski
Faculdade de Engenharia/Unesp - Av. Brasil Centro, 56
15385-000 Ilha Solteira - SP
Heraldo Nélio Cambraia
Departamento de Mecânica - Centro Politécnico - UFPR
81531-990 Curitiba - PR

Abstract. *Although reconstruction and prediction of time series have been studied for a number of theoretical nonlinear systems, the literature lacks results for real-life situations. In this work, we have embedded some experimental time series, obtained from the wake of the flow past a rectangular cylinder, into a state space employing delay coordinates. After performing noise reduction, by means of singular value decomposition, we have generated the induced nonlinear mapping using a local approximation, therefore allowing for short-term predictions of the future behavior of the system, with information based only on past values. In addition, an error estimate for this problem has been included.*

Keywords: *Chaotic time series, Short-term prediction, Rectangular cylinder wake*.

1. INTRODUCTION

The dynamical analysis of nonlinear physical systems is an important inter-disciplinary subject. Its objective is to extract qualitative and quantitative information from experimental process through observation of one or more time series. There are several algorithms for the analysis of chaotic time series, as the proposed by Brown (1991), Eckmann (1985), Eckmann (1992), Farmer (1983), Grassberger (1983) and Hentschel (1983). The purpose of these algorithms is to calculate geometric and dynamic invariants of an underlying strange attractor, an interesting alternative approach would be to predict the behavior of the system, following Abarbanel (1990), Farmer (1987) and Casdagli (1989).

Although reconstruction and prediction of time series have been studied for a number of theoretical nonlinear systems, the literature lacks results for real-life situations. In this work, following Farmer (1987) and Casdagli (1989), we have embedded some experimental time series, obtained in the wake of the flow past a rectangular cylinder, into a state space employing delay coordinates. After performing noise reduction, by means of singular value decomposition, we have generated the induced nonlinear mapping using a local approximation.

2. EXPERIMENTAL FACILITY AND METHODS

2.1. Water Tunnel

The experimental tests have been conducted in an open-circuit vertical water tunnel driven by gravitational action. The water tunnel has been employed in the continuous mode of operation, in which the flow control valves and the filling system valve are adjusted in order to keep constant the water level inside the upper reservoir. This mode allows the conduction of tests with long duration, constant free stream velocity, and turbulent level of less than 1%. For further details concerning the facility, one should refer to Vieira (1997).

2.2. Test Model

During the tests, a rectangular cylinder of dimensions $3{,}01 \times 5{,}99 \times 146{,}5$ mm ($W \times H \times L$) has been employed. The model has been machined, grinded and polished in order to achieve good dimensional quality, sharp edges and smooth surfaces. The test model has been firmly attached to the test section rear window with null incidence angle, that is, with the smaller edge (W) facing upstream.

2.3. Experimental Technique

For the instantaneous velocity measurement, it has been employed a constant temperature anemometer Dantec® StreamLine model 90N10, employing a CTA 90C10 module and a fiber probe model 55R11, connected to a computer-based data acquisition system. The anemometer is controlled through the application software StreamWare version 1.11, supplied by the manufacturer.

The probe, installed on an L-shaped support model 55H22, has been introduced in the test section through one of its lateral windows and placed in the cylinder wake. The probe support has been attached to a positioning device, and the probe distance from the test model have been settled to $5H$ downstream and approximately $2H$ sidewise.

The velocity signal, without passing through filters, has been discretised and recorded using a sampling rate of 1kHz. A typical record of sampled data had 2×10^3 data points.

2.4. Dimensionless Parameters

A dimensionless number that exerts a major influence on the flow around cylindrical bodies is the Reynolds number, here defined as

$$Re = \frac{VW}{\nu} \tag{1}$$

where V is the free stream velocity, W is the cylinder characteristic diameter (in the present work, the cross section width) and ν is the fluid cinematic viscosity. The Reynolds number uncertainty has been estimated to be within ±5%.

The cylinder side ratio, given by the relation between the height and width of its rectangular cross section, has been fixed in 2. The cylinder aspect ratio, defined as the relation between its length L and characteristic diameter W, has been found to be 24. The geometric blockage ratio, given by the relation between the cylinder frontal area and the test section transversal area, has been kept under 4,9%, so that no correction procedure has been employed.

3. NUMERICAL METHODS

3.1 Method of Delays

To perform noise reduction and short-term prediction, we have started by embedding a single time series in a state space. Following the approach introduced by Packard *et al*. (1990) and established by Takens (1991), if $\dot{\mathbf{Y}} = F(\mathbf{Y})$ spans a *d*-dimensional flow, then

$$\mathbf{x}_i(t) = \{x_i(t), x_i(t+\tau), \cdots, x_i(t+[m-1]\tau)\} \tag{2}$$

where x_i is an arbitrary component of **Y,** τ is a adequate time delay and *m* is the embedding dimension, so that Eq. (2) provides a continuous embedding space for that flow. The trajectories on both spaces are topologically equivalent, although the reconstructed attractor is not identical to the original one, that is to say the dimension of the reconstructed state space is not necessarily equal to the dimension of the real state space that represents the dynamic physical system.

In the absence of noise, any time delay may be selected; however real time series are contaminated by noise, and the reconstruction depends on the adequate choice of time delay. In this work, we have used a time delay of the order of the autocorrelation time from the time series (Fiedler-Ferrara, 1994).

The choice of the embedding dimension *m* has been performed by using the singular value decomposition of the covariance matrix, as it will be explained in the next section.

3.2 Noise Reduction

The application of the method of delays to the time series of N_T data points results in a sequence of $N=N_T-(m-1)$ vectors in the embedding space. This sequence can be used to construct a trajectory matrix $\mathbf{X}_{Nxm}$,

$$\mathbf{X} = N^{-1/2}\begin{bmatrix}\mathbf{x}_1^T \\ \mathbf{x}_2^T \\ \vdots \\ \mathbf{x}_N^T\end{bmatrix} \tag{3}$$

The covariance matrix of the components of $\{\mathbf{x}_i\}$, averaged over the entire trajectory, can be defined by

$$\Xi = \mathbf{X}^T\mathbf{X} = \frac{1}{N}\sum_{i=1}^{N}\mathbf{x}_i\mathbf{x}_i^T \tag{4}$$

which is a real and symmetric $n \times n$ matrix, and therefore it can diagonalized by singular value decomposition, $\Xi = UAU^{-1}$,where A is a diagonal matrix. The elements in the diagonal of matrix A are the eigenvalues of Ξ and the columns of U are its normalized eigenvectors. This transformation of variables is called Singular Value Decomposition (SVD) and represents the attractor on a space where the state variables are statistically independent.

The eigenvalues can be computed numerically and the embedding dimension is estimated by counting the non-null eigenvalues. However, the presence of noise in the time series can turn all the eigenvalues non-null. In Broomhead (1986) and Cambraia (1997), a threshold value has been established for choosing the embedding dimension and for eliminating white and Gaussian noises, obtaining good results.

3.3 Local Approximation

Following Casdagli (1989) and Farmer (1987), let $\tilde{f}:\Re^m \to \Re^m$ be a smooth map with a strange attractor α, let $x_n = \tilde{f}^n(x_0),\ 1 \le n \le \infty$ be a sequence of iterates under $\tilde{f}$ lying on α. The problem of nonlinear prediction of this sequence is to construct a smooth map $f_N:\Re^m \to \Re^m$ using $x_n, 1 \le n \le N$, where $x_{n+1} = f_N(x_n),\ 1 \le n \le N-1$. For $m > 1$, this problem amounts to fitting m smooth functions $\pi_i f_N:\Re^m \to \Re$ through the data points $(x_n, \pi_i x_{n+1}),\ 1 \le n \le N-1$ for $i = 1, \cdots, m$, where π_i denotes the projection onto the i^{th} coordinate.

To construct the local predictor, the graph of $\pi_i f$ is constructed by piecing together local graphs. If the value of $\pi_i f$ is required at a point *x*, the *k* nearest neighbors of $x_1, \ldots, x_{N-1}$ to *x* are found, and a polynomial of degree at most *d* (*d* small) is fitted, by least squares method, through the corresponding data points, for k at least equal to $(m+d)!/m!d!$, for the solution of the least squares problem. As a preliminary study, it has been used *k*=1, the zero-order approximation.

An error estimate for the local prediction has been computed by the root-mean-square error, given by $\sigma(T) = \left\langle [x_{pred}(t,T) - x(t+T)]^2 \right\rangle^{1/2}$, normalized by the RMS deviation of the data, computed as $\sigma_x = \left\langle (x - \langle x \rangle)^2 \right\rangle^{1/2}$.

4. RESULTS

In this work, the experimental time series has been obtained by measuring the velocity in the wake of a rectangular cylinder with a side ratio *H*/*W* = 2, for Reynolds numbers equal 100 and 4000.

Figure 1 and 2 present the results obtained from noise reduction for Reynolds numbers equal 100 and 4000, respectively. Figure 1(a) shows the original data, while Fig. 1(b) presents these data after the noise reduction performed by SVD, and Fig. 1(c) displays the results for noise reduction performed by a moving-average filter with 100 neighboring points. It can be seen in Fig. 1(b) the relative smoothness of the curve and in Fig. 1(c) a better result obtained by moving-average filter.

The left column in Fig. 2 shows the time series and the right, the density power spectra. Figures 2(a, b) show the original data, Fig. 2(c, d) show the results after noise reduction performed by SVD and Fig. 2(e, f) show the results for a moving-average filter with 100 neighboring points. Due to the fact that SVD transformation of variables represents the attractor on a space where the state variables are statistically independent, the choice of the most significant eigenvalues rejects the portion of the inherent experimental noise. It can be seen in Fig. 2(c) the relative smoothness of the velocity signal, and in Fig. 2(d), the lower level of power for the higher frequencies. As one can see in Fig. 2(e, f), the conventional moving-average filter seems not adequate to treat this kind of data, which has a lot of information distributed over a great range of frequency.

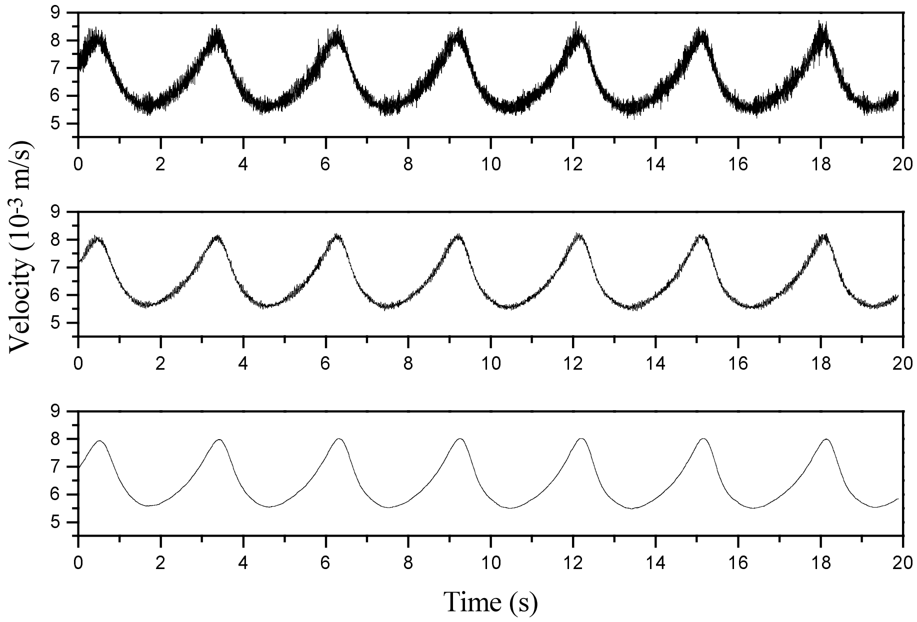

Figure 1: Results of noise reduction in a experimental data for *Re*=100.

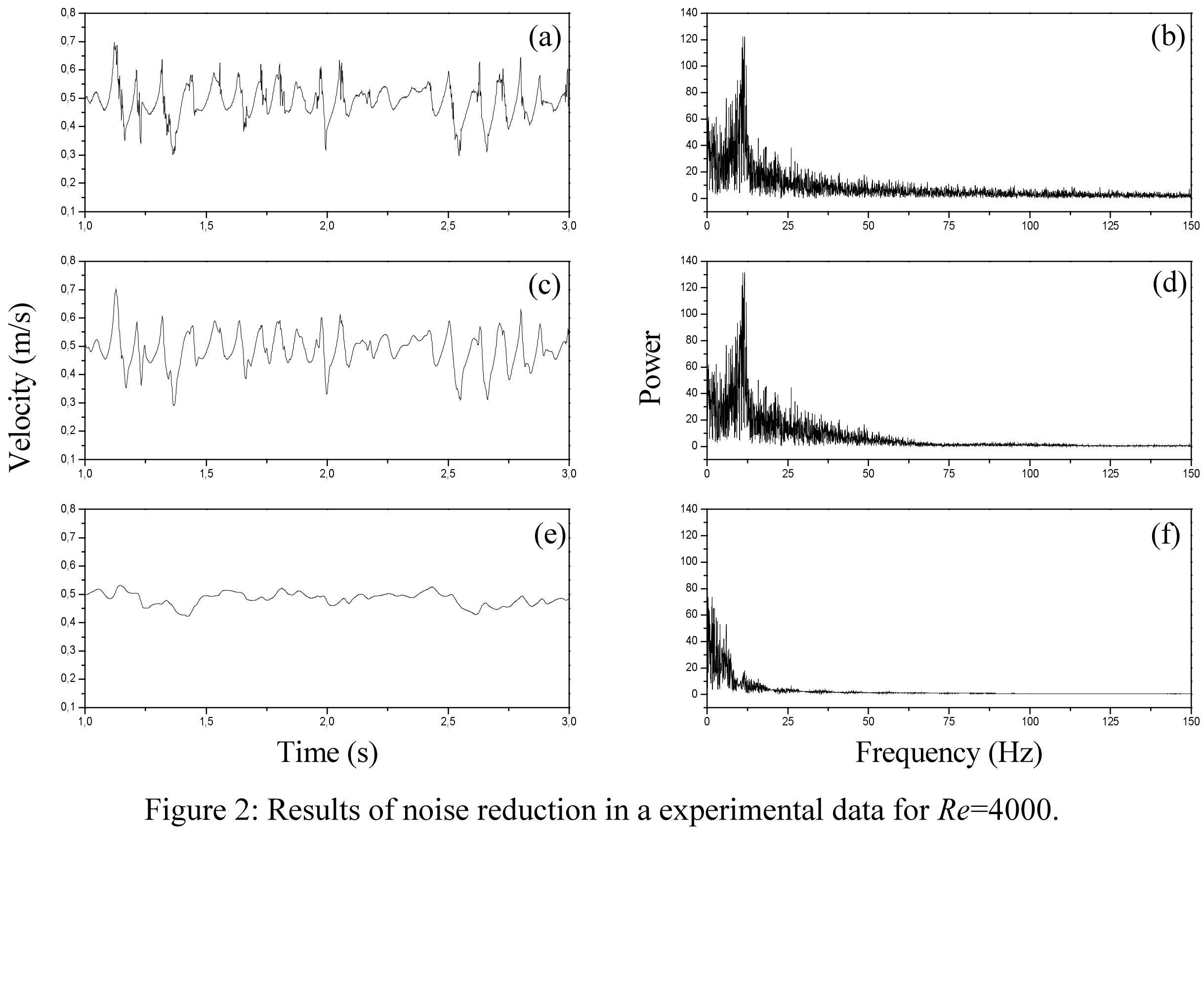

Figure 2: Results of noise reduction in a experimental data for *Re*=4000.

Figure 3 shows the plots of the trajectory **X** projected onto mutually orthogonal planes spanned by the first three singular vectors {$\mathbf{c_1}$, $\mathbf{c_2}$, $\mathbf{c_3}$}. The i^{th} point on the ($\mathbf{c_j}$, $\mathbf{c_k}$)-plane is given by $(\mathbf{c}_j^T \mathbf{x}_i, \mathbf{c}_k^T \mathbf{x}_i)$. It can be seen that the attractor is indeed a limit cycle for *Re*=100.

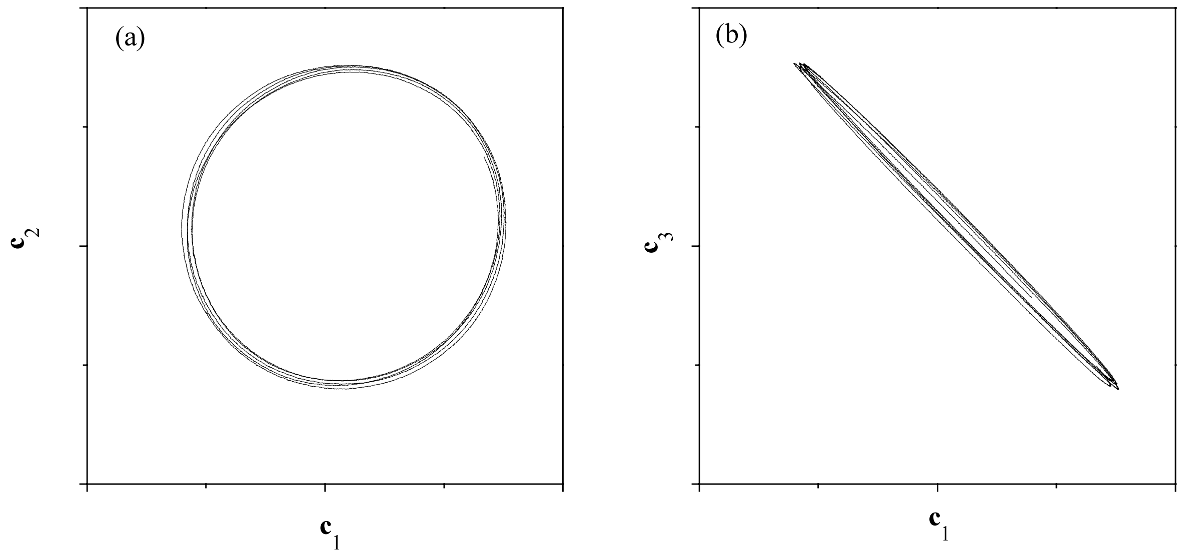

Figure 3: Plots of the trajectory **X**, projected onto two orthogonal planes obtained by SVD.

Figure 4 presents the normalized error for Reynolds number 4000. As one can see, the normalized error has a quasi-exponential behavior. According to Farmer (1987), for chaotic signal this error has a exponential increase with the time prediction, and further, the quality of the local approximation depends on several parameters, including the prediction time, the number of data points, the attractor dimension, the metric entropy, and the noise-to-signal ratio (NSR). If the typical space between data points is about the NSR, then the prediction is limited by noise, and if the typical space is greater than NSR, the accuracy of prediction is limited by the number of data points.

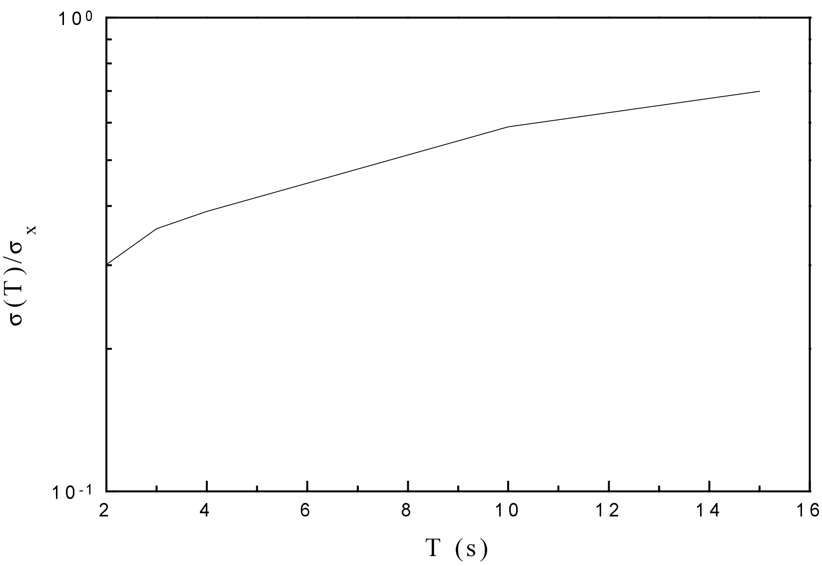

Figure 4: Normalized error estimate versus prediction time for 3000 prediction points.

5. CONCLUDING REMARKS

It has been made a preliminary study of a time series obtained from velocity measurements in the wake of a rectangular cylinder for Reynolds number of 4000. The study has been performed by using concepts from theory of dynamical systems.

Although some evidences of chaotic behavior have been found, like broadband Fourier spectra, irregularity and a near-exponential behavior of the normalized error of the local approximation, it remains to be studied the characteristic Lyapunov exponents, entropy, correlation dimension and higher order local approximations.

Acknowledgements

The authors wish to acknowledge the financial support provided by FAPESP, *Fundação de Amparo à Pesquisa do Estado de São Paulo* (grant Nos.98/00982-6, 97/12818-3 and 97/12249-9).

REFERENCES

Abarbanel, H. D. *et al.,* 1990, Prediction in chaotic nonlinear systems: Methods for time series with broadband Fourier spectra, *Physical Review A,* v.41(4), pp.1782-1807.

Broomhead, D. S. & King, G. P., 1986, Extracting quantitative dynamics from experimental data, *Physica D,* v.20, pp.217-236.

Brown, R. *et el*, 1991, Computing the Lyapunov spectrum of a dynamical system from an observed time series, *Physical Review A*, v.43(6), pp.2787-2806.

Cambraia, H. N., 1997, Noise reduction in time series by using singular value decomposition, *in*: CD-ROM Anais do *XIV Congresso Brasileiro de Engenharia Mecânica*.

Casdagli, M., 1989, Nonlinear prediction of chaotic time series, *Physica D,* v.35, pp.335-356.

Eckmann, J-P. & Ruelle, D., 1985, Ergodic theory of chaos and strange attractors, *Rev. Mod. Phys.*, v.57, pp.617-656.

Eckmann, J-P. & Ruelle, D., 1992, Fundamental limitations for estimating dimensions and Lyapunov exponents in dynamical systems, *Physica D,* v.56, pp.185-187.

Farmer, J. D. *et al.,* 1983, The dimension of chaotic attractors, *Physica D,* v.7, pp.153-180.

Farmer, J. D. & Sidorowich, J. J., 1987, Predicting chaotic time series, *Physical Review Letters,* v.59(8), pp.845-848.

Fiedler-Ferrara, N. & Prado, C. P. C., 1994, *Caos*: uma introdução, 1.ed., São Paulo: Edgard Blücher Ltda, 402p.

Grassberger, P. & Procaccia, I., 1983, Measuring the strangeness of strange attractors, *Physica D*, v.9, pp.189-208,.

Hentschel, H. G. E. & Procaccia, I., 1983, The infinite number of generalized dimensions of fractals and strange attractors, *Physica D,* v.8, pp.435-444.

Packard, N. H. *et al.,* 1980, Geometry from time series, *Physical Review Letters,* v.45, pp.721-719.

Takens, F., 1981, Detecting strange attractors in turbulence, in: Dynamical systems and turbulence, Lectures notes in Math. 898, Springer, pp.366.

Vieira, E.D.R., 1997, Estudo qualitativo e quantitativo de escoamentos ao redor de corpos não-aerodinâmicos utilizando-se técnicas de visualização em meio hidrodinâmico, Tese de doutorado, ITA, São José dos Campos, 200p.

Williams-Stuber, K. & Gharib, M., 1990, Transition from order to chaos in the wake of an airfoil, J. Fluid Mech., v.213, pp.29-57.